A Measuring Approach in Bidding Selection Procedures for Large Projects

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Abstract

The problem of selecting a credible bidder during tendering tends to haunt both the client and the service provider. Even though this problem is rife in many countries globally, it is especially true in developing countries. Many bidders do their best in developing exceptional proposals, but fairness on the side of the client in selecting a deserving winner is not always the applied approach. There is a growing demand for the services of consultants due to global competition and rapidly advancing technology. Competition is tough for consultants as they strive to displace one another. The problem of selection of a worthy consultancy by client is such that often, the consultants do not know how they are selected, and the client usually cannot demonstrate by any systematic proof that the selected consultant is the best one. The procedure and criteria for selection are “not so clear”, and there is usually no transparency in the selection process. This paper discusses three scientific methods proposed for use to select a criterion-based worthy bidder in management consulting. Mathematical and statistical methods are used to offset the bias/subjectivity in the selection process. Attributes are defined to address both added value and reduced costs. These are then built-in in the selection process. A matrix is presented and transformation methods used to derive useful descriptive statistics (scores) for selecting the best bidder. An example is used for illustration and a closing discussion ends the presentation.

Keywords: Quantitative measures, Attributes, Criteria, Scientific method, SAW, WP, TOPSIS.

Introduction

Many clients who tend to involve consultants in their organisations, even though they would deny, usually have (at least) two possible severe problems. First, they tend to favour a particular consultancy (bias) for no merit reason. Secondly, they may even (genuinely) lack the knowledge to select a worthy consultancy that is capable of ensuring added value and reduced costs to their organisation(s). Transformation methods (in mathematics sense) are sometimes used as scientific instruments for solving decision problems ‘optimally’. These methods are explored in this article to address decision problems that require scientific approaches such as scoring methods for comparing two or more alternatives in order to enhance objectivity and reduce bias in the final selection. In general, according to Triantaphyllou [1], decision problems are based on several variables and numerous conditions. This makes it easy for tricksters to manipulate the systems used. However, even in the most complex of circumstances, there are always scientific methods to deal with any situation requiring problem solving [2]. Decision problems that are addressed using methods that are based on subjective criteria and favoritism (or any kind of bias) tend to be controversial in the public eye and to those who have interest in addressing the problem [3,4]. When these are done in a competitive context, losers tend to question the methods and criteria used, and the merits of the procedures followed. Therefore, members in the same team may end up giving inconsistent explanations for the same procedures when subjectivity and bias were dominant. On the other hand, scientific approaches are reputable for reducing (or sometimes eliminating) bias and defying subjectivity [5]. Without doubt, fairness is necessary in any competitive situation. Systematic methods, such as scientific ones, are capable of offsetting bias in the selection process. This paper presents selection process using scientific method during tendering/bidding for a project. The terms ‘tenderer’ and ‘bidder’ mean the same thing in the presentation, which is an individual, a group or a party that has submitted a proposal to be chosen to win a project in a consulting situation.
Rationale and Context
The article intends to advocate fairness in selection of a bidder in tendering, and to provide methods to ensure that bias is reduced or eliminated. Three quantitative methods are presented in this paper, which are used to work together in selecting a winning tenderer from a number of alternative bidders. Ideally, the three methods should be used together and have to demonstrate consistency in the selection of the winner of a bidder. On aggregate, the methods should produce the winning tenderer that (among other conditions):

- Is satisficing (i.e. meets the minimum requirements);
- Has the highest score for order of preference;
- Resembles the ideal features best; and simultaneously
- Contrasts the undesirable features most.

The discussion hopes to influence tendering from subjective judgement in selecting a consultant who wants to provide the service from a number of other consultants who seek to be selected for the same job(s). Desirable attributes and criteria for use in the process are proposed. They should be clear and conspicuous to the bidders.

Methods
The methods are SAW (simple additive weight), WP (weighted product) and TOPSIS (technique for order preference by similarity to ideal solution). The principles of these methods, according to Hwang et al. [6], are:

- SAW, TOPSIS and WP assign scores/values to compare alternatives
- SAW and WP assign scores to an alternative (competitor) with the highest score indicating highest preference
- TOPSIS is a score derived from two distance measures such that the chosen solution is “as close to the positive ideal solution (PIS) as possible and as far away from the negative ideal solution (NIS) as possible.

Notation and Setting
Based on the premise that decisions are characterized by many attributes (m, say), the exercise is in a scenery of Multiattribute Decision Making (MADM). The different attributes are of unequal importance, and “weights” are used to signify the differences in the worth of each attribute. Naturally, attributes can be classified as benefit attributes and cost attributes.

Let \( J = \{ \text{benefit attributes} \}; I = \{ \text{cost attributes} \}; \ A_i, \ i = 1, 2, \ldots, n, \) be the \( i^{\text{th}} \) company (or bidder); \( X_j, \ j = 1, 2, \ldots, m, \) the \( j^{\text{th}} \) attribute used in the selection (therefore becoming a criterion); and \( x_{ij} \ (> 0) \) the observed scale value of the \( j^{\text{th}} \) attribute of the \( i^{\text{th}} \) alternative.

Multidimensional forms are easily presented by a matrix form. Define the \( n \times m \) matrix \( X \) with columns representing the criteria and the rows representing the companies. (The companies are the alternatives or the bidders in competition.) This matrix is commonly known as the decision matrix [7] and has the form:

\[
X = \begin{bmatrix}
    x_{11} & x_{12} & \cdots & x_{1m} \\
    x_{21} & x_{22} & \cdots & x_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{n1} & x_{n2} & \cdots & x_{nm}
\end{bmatrix} \tag{1}
\]

Nature presents different attributes in different units while known methods are applicable only on units that are incommensurate or incommensurable (incomparable). It becomes necessary in general, therefore, to converts scores into units that are commensurate (i.e. comparable). Yoon and Hwang [8] define vector normalisation of matrix elements as:

\[
p_{ij} = \frac{x_{ij}}{\sum_{s=1}^{m} \sum_{k=1}^{n} x_{sk}} \tag{2}
\]

The scores defined by (2) are performance indexes [7] and have the following two properties:

1. \( 0 \leq p_{ij} \leq 1 \); and
2. \( \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} = 1 \).

Define the matrix \( P \) of normalised elements (called performance index matrix) as:

\[
P = \begin{bmatrix}
    p_{11} & p_{12} & \cdots & p_{1m} \\
    p_{21} & p_{22} & \cdots & p_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    p_{n1} & p_{n2} & \cdots & p_{nm}
\end{bmatrix} \tag{3}
\]

The term “optimise” (or “optimum”) could imply “minimise” or “maximise” (“minimum” or
maximum”), depending on the nature of the problem/question. This means that the operation to “optimise” shall be used for the appropriate/required purpose. Bellman and Zadeh [9], Hwang et al. [6], Ozelkan and Duckstein [10], Seo and Sakawa [11], Steuer [12], White [13], Yu [14] and Zeleny [15], among others, represent MADM problems using this operator in the following approach:

\[
\text{optimise} = \left[ f_1(x) \quad f_2(x) \quad \ldots \quad f_m(x) \right] \quad (4a)
\]

such that

\[
x \in X = \{ x : g_h(x) \leq, =, \geq 0; h = 1, 2, \ldots, k \} \quad (4b)
\]

where

\[
f_i(x): \text{benefit objective for maximization, } j \in J \quad (4c)
\]

\[
f_i(x): \text{cost objective for maximization, } j \in J \quad (4d)
\]

and \(X\) is a set of appropriate constraining functions \(g_h(x)\).

Equations (4) above, denote the measure to be optimised, where the problem consists of a total of \(n\) decision variables, \(k\) constraints and \(m\) attributes. The presentation in this paper illustrates with SAW, WP and TOPSIS to address (4a). To ease the discussion explains these measures.

**Reference Points**

Hwang et al. (1993) define reference points from equation (4) for \(j = 1, 2, \ldots, m\) as follows:

\[
f_i^* = \begin{cases} 
\max_{x \in X} f_j(x) & \forall j \in J \\
\min_{x \in X} f_j(x) & \forall i \in I 
\end{cases} \quad (5)
\]

and

\[
f_i^\wedge = \begin{cases} 
\min_{x \in X} f_j(x) & \forall j \in J \\
\max_{x \in X} f_j(x) & \forall i \in I 
\end{cases} \quad (6)
\]

Define \(f^* = (f_1^* \quad f_2^* \quad \ldots \quad f_n^*)\) as the solution of (5) which consists of individual best feasible solutions for all objectives. Similarly, define \(f^\wedge = (f_1^\wedge \quad f_2^\wedge \quad \ldots \quad f_n^\wedge)\) as the solution of (6) which consists of individual worst feasible solutions for all objectives.

According to Hwang et al. [16], \(f^\wedge\) is the positive ideal solution (PIS) and \(f^\wedge\) is the negative ideal solution (NIS) and these are always out of the feasible region of (4a) and (4b). This setback is offset by a compromise approach later in section 6.3 of descriptive measures when TOPSIS is discussed.

**Descriptive Measures**

**SAW**

Dawes [17] explains that SAW assigns scores to alternative competitors. To assign a score for an alternative, the process requires the contribution of each attribute, which is obtained by multiplying the weights (signifying the level of importance) with the corresponding performance indexes. The resulting score is obtained by adding contributions from each alternative.

**WP**

WP also assigns scores to alternatives [17]. Yoon and Hwang [8] explains that to assign a score for an alternative, the process requires the contribution of each attribute, which is obtained by multiplying the attribute values together, with the corresponding weights as exponents. A positive exponent is assigned for benefit attribute and a negative one for cost attribute. This is because value is directly proportional to benefit and inversely proportional to cost.

**TOPSIS**

TOPSIS reduces a k-dimensional objective space to a 2-dimensional (the distance from PIS vs. the distance from NIS) objective space, and hen removes the inherent incommensurability between the original variables. There is a “conflict” between the distances from PIS and that from NIS because we cannot simultaneously obtain the shortest distance from PIS and the longest distance from NIS [7]. As a result compromise criteria to replace these ones were introduced. These compromise criteria are such that:

- “The shortest distance from PIS” is substituted by “as close to PIS as possible”; and
- “The farthest distance from NIS” is substituted by “as far away from NIS as possible”.

The latest criteria are fuzzy (vague or unclear), but positive for business/industry because of emerging similarities as follows:

- “As close to PIS as possible” is similar to “as much profit as possible”; and
- “As far away from NIS as possible” implies “as little risk as possible”.

The solution proposed for equation (4) is also a compromise solution of the original \(m\)-attribute problem.
Transformation Methods: Calculation of Weights

It was pointed earlier that s generally, attribute differ in their level of importance, some attributes could be more important than others. This paper uses weights \( w_j, j = 1, 2, \ldots, m \) to denote the relative importance of attributes, where larger weights implies higher level of importance. It means that a more important attribute is shown by a larger value of the weight. A measure called entropy is used in this paper to determine the weights. The next section starts a discussion on the entropy.

Entropy

Entropy is a measure to measure the amount of disorder in a system [7,18,19]. In statistical mechanics, entropy is essentially a measure of the number of ways in which a system may be arranged, or a measure of "disorder" implying that the higher the entropy, the higher the disorder [20,21]. According to Atkins and De Paula [22], entropy quantifies, in the sense of an expected value, the information contained in a message. Shannon and Weaver [23] applied entropy for measuring the relative contrast intensities of performances using \( P \) to represent the average intrinsic information transmitted to the decision maker. The relative performance of an alternative indicated in given data carries weight for he analysis used. In fact, entropy is a measure of certainty/uncertainty in the information formulated according to probability theory. It indicates the amount of decision information that each performance index contains [7]. It indicates that a broad distribution represents more uncertainty than doe a sharply peaked one.

Let \( k = \frac{1}{\ln n} \), where “\( \ln \)” denotes the natural logarithm to the base e. Deng et al. [7] define the entropy measure \( e_j \), as:

\[
e_j = -k \sum_{i=1}^{n} p_{ij} \ln p_{ij}, \quad j = 1, 2, \ldots, m (7)
\]

Axiom 1

\( 0 \leq e_j \leq 1 \)

Degree of Divergence

The degree of divergence \( (d_j) \) represents the inherent contrast of the attribute \( X_j \) [7,15]. It is defined (and thus calculated) as:

\[
d_j = 1 - e_j, \quad j = 1, 2, \ldots, m (8)
\]

When performance ratings diverge more for the attribute \( X_j \), then \( d_j \) becomes larger. This means that the attribute \( X_j \) is more important for the problem at hand [24]. By using the definition of entropy and equation (7), the degree of divergence is shown to be the measure of the amount of order of an attribute.

Axiom 2

\( 0 \leq d_j \leq 1 \)

Further logic about the measure \( d_j \) is that an attribute is not important for a specific problem if in that problem all the alternatives have identical performance ratings for that particular attribute. Therefore, if all performance ratings for that attribute are the same, the attribute can be eliminated for the situation on which a decision is based as it transmits no information to the decision maker.

Weights

The weights derived from entropy (through the degree of divergence) for each attribute are given by Deng et al. [7] as:

\[
w_j = \frac{d_j}{\sum_{k=1}^{m} d_k}, \quad j = 1, 2, \ldots, m (9)
\]

Axiom 3

\( 0 \leq w_j \leq 1 \)

Transformation Methods

Saw Index

Consider the \( n \times m \) matrix of performance \( P = (p_{ij}) \), and the \( m \)-vector of weights given by the column vector \( w = (w_1, w_2, \ldots, w_m)^T \). (Here the superscript \( T \) denotes the transpose of matrix/vector.) Define the transformation \( V_1 : X = (Pw) = (v_1, v_2, \ldots, v_n)^T \), a \( n \)-column vector, where:

\[
v_j = \sum_{i=1}^{n} w_j p_{ij}, \quad i = 1, 2, \ldots, n (10)
\]

These values are the SAW scores to be compared. For this purpose, the next discussion order the values defined in (10) from largest to smallest, that is, \( v_{(i)} > v_{(j)} \) if \( i < j \). The ordered SAW scores
are denoted by $v_{(i)}, v_{(2)}, \ldots, v_{(n)}$. Since the scores are intended to measure the “worth” of a competitor relative to other competitors, where the alternative competitors are denoted by the letter A (with appropriate sub- and superscripts as necessary), when these competitors are used in the place of their scores, the ordered list shall appear as:

$$A^{(m)}_{(1)} A^{(m)}_{(2)} \ldots A^{(m)}_{(n)}$$

Here, $A^{(m)}_{(i)}$ denotes the $i^{th}$ ranked (bidding) competitor in terms of the SAW measure.

**WP Index**

Recall $\omega = (w_1, w_2, \ldots, w_n)^T$. Define the transformation function $V_2: X = V_s$, where the $n$-row vector $V_s = (v_{1}^{(wp)} \ v_{2}^{(wp)} \ \ldots \ v_{n}^{(wp)})$ is defined by:

$$v_i^{(wp)} = \prod_{j=1}^{m}(x_{ij})^{w_{j}}, i = 1, 2, \ldots, n$$

The main frustrating problem with these values is that they do not have definite bounds, such as a lower bound. As such they lack an appealing interpretation guideline. To offset this, define the ideal vector $f^* = (f_1^* \ f_2^* \ \ldots \ f_n^*)^T$, where $f_j^*$ is the individual best option from the $j^{th}$ attribute. (Section 8.3 calls this a positive ideal solution, or PIS.)

Define

$$v_{ideal} = \prod_{j=1}^{m}(f_{j})^{w_{j}}, \quad i = 1, 2, \ldots, n$$

Finally, define the index value to be used in the WP method as

$$R_i = \frac{v_i}{v_{ideal}}, \quad i = 1, 2, \ldots, n \quad (11)$$

**Axiom 4**

$$0 \leq R_i \leq 1 \text{ for all } i = 1, 2, \ldots, n$$

The larger $R_i$ index value of an alternative competitor implies that the (anticipated) performance of that competitor would be better. To go ahead with ordering, the WP scores are ordered from largest to smallest, and the ordered WP scores are $R_{(1)}, R_{(2)}$, \ldots, $R_{(n)}$. The corresponding competitors listed in the orders given for the scores are written:

$$A_{(1)}^{(wp)} A_{(2)}^{(wp)} \ldots A_{(n)}^{(wp)}$$

**TOPSIS Index**

In section 5 it was reported that the PIS $f^*$ and the NIS $f^*$ are always out of the feasible region of (4a) and (4b). As a result TOPSIS is a compromise statistic derived from some combination of NIS and PIS. The TOPSIS index is constructed [7,8] from distance measures as follows:

$$S_i = \left[ \sum w_j (f_j - x_{ij})^2 \right]^{\frac{1}{2}}, \quad i = 1, 2, \ldots, n \quad (12)$$

and

$$s_i = \left[ \sum w_j (x_j - f_j^*)^2 \right]^{\frac{1}{2}}, \quad i = 1, 2, \ldots, n \quad (13)$$

The TOPSIS index is an overall performance measure calculated for each alternative $S_i, i = 1, 2, \ldots, n$ by:

$$T_i = \frac{S_i}{S_i + s_i}, \quad i = 1, 2, \ldots, n \quad (14)$$

**Axiom 5**

$$0 \leq T_i \leq 1 \text{ for all } i = 1, 2, \ldots, n$$

Again, the larger $T_i$ index value of an alternative competitor implies that the (anticipated) performance of that competitor would be better. When ordering, the TOPSIS approach also orders scores from largest to smallest. The ordered TOPSIS scores are $T_{(1)} , T_{(2)} , \ldots , T_{(n)}$. The corresponding competitors appear as:

$$A_{(1)}^{(topsis)} A_{(2)}^{(topsis)} \ldots A_{(n)}^{(topsis)}$$

**Remark**

Selection of a bidder that are for small projects and limited funds may be allowed to depend on any single methods since the tendency in South Africa is to not follow the tender root because of its complications. Use of one method is still simple but with increased complexity, but the positive aspect is that it incorporates a scientific method. However, in the selection of a bidder in a large project situation, use of any one of these may still lead to ideal selection. It is therefore recommended to be guided by a reconciliation of the results of all the three methods. Uniformity in
the guidelines given by the three methods should guide the final selection. A numerical example follows.

**Numerical Example**

The next set of attributes were defined and adopted in a workshop held by consultant and client groups (addressing ‘very large & expensive’ project selection problems) in M-SM Hotel in 08-12 November 1999 [25].

**Project Background**

A division of government department in Mpumalanga Province, South Africa, initiated activities that would enhance its processes for accelerating service delivery in its operations. New ways of doing things were found to be necessary, and new tasks would imply that there would be a need to find help from outside. Also, there was going to be many planned tasks which needed skills and experiences that the department seemed to lack. Skills were to be found from somewhere outside the department.

The division decided that external consultants would be used. A total of nine common attributes were specified and used in the invitation of consultants. In this paper these are stated as $X_1$, $X_2$, $X_3$, $X_4$, $X_5$, $X_6$, $X_7$, $X_8$, $X_9$. Eight of these were benefit attributes while the eighth one was a cost attribute. Certain minimum standards were set that needed to be met for each attribute by the bidding consultants in order for them to qualify for the project(s) required.

**Invitation to Tender**

The division then prepared all the required requisitions inside the department and then invited consultants to bid/tender for the job. Ten consultants tendered for the job. Four of these failed to meet the minimum requirements in some of the nine attributes specified. They were disqualified immediately and were duly informed about the outcomes and the details of the results. Six who remained left were allowed to compete.

**Further Evaluation of Tenders**

The remaining six consultants were named/denoted $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$ to ensure that their identity was unclear to the evaluators of the proposals. A $6 \times 9$ matrix was formed in which the six rows represented the bidders and the nine columns were the attributes. Five experienced experts who knew the tasks needed were appointed and briefed on their role to select the proper best bidder. (Apparently the evaluators did not even know about the project until they were asked to evaluate, which made them unaware of the bidders as well.) The division representatives and the evaluators came with a model to award marks according to the attributes on a 100% scale.

To make the evaluation consistent, each of the five would take the six documents for the bidders and mark them within two days while staying in a Nelspruit hotel. The evaluators were not allowed to communicate, and they were not expected to make calls to department members until they had done all the marking. These experts were only allowed to call their families using the telephone in the hotel. They all completed the tasks within one day (as they worked throughout the night as well). They called the division representative they were given and she collected all the marked work and a meeting was convened for the way forward.

**Consolidated Evaluations**

Marks for each attribute were compared and it was agreed that they were consistent. The average marks for each attribute were then given for each bidder. A matrix was then designed presenting these average marks for each attribute per bidder.

The result obtained, remembering that the eighth column is a cost attribute while others are benefit attribute, was as follows:


**Determining Scores**

The three methods to be used in determining scores for comparison of the bidders are SAW, WP and TOPSIS. A statistician was employed to carry out the calculations. She did the min Excel. The statistician advised that the scores and values should retain decimals to avoid errors that may emerge due to rounding off. A minimum of four decimals was agreed upon, but longer ones were also permissible.

**Performance Matrix**

These methods (SAW, WP, TOPSIS) require a performance matrix. It was calculated and it is:
Consistency is shown, generally, but TOPSIS interchange the order of A, and A, while in both SAW and WP show A to be leading
while $A_4$ follows it. There is no doubt though, that $A_1$ should be the chosen bidder.

**TOPSIS and PIS as Individual Scoring Measures**

TOPSIS is a compromise measure combining NIS and PIS, which are independent measures by themselves. This section determines the pattern of results that would emerge if NIS and PIS were to feature as independent measures.

**NIS**

NIS prefers largest distances. Taken from the above calculations the following are obtained:

\[
s_j, \quad j = 1, 2, \ldots, 6
\]

| $s_j$ | 38.1675 | 26.4822 | 29.9674 | 19.2975 | 14.3787 | 18.5974 |

From largest distance to the smallest one they are:

$s_1, s_3, s_2, s_4, s_6, s_5$

These would make the order of preference:

$A_1, A_3, A_2, A_4, A_6, A_5$

**PIS**

On the other hand PIS prefers smallest distances. Again, taking appropriate values from previous calculations the following are obtained:

\[
s_j, \quad j = 1, 2, \ldots, 6
\]


From smallest to largest distances they are:

$s_1, s_3, s_2, s_4, s_6, s_5$

These would make the order of preference:

$A_1, A_3, A_2, A_4, A_5, A_6$

**Summary of NIS and PIS**

NIS $A_1, A_3, A_2, A_4, A_6, A_5$

PIS $A_1, A_3, A_2, A_4, A_5, A_6$

NIS and PIS are consistent in the order of the first four bidders, and the positions of the last two are interchanged. There is consistency in these measures in that the leading bidders are in exactly the same positions and the two bidders are at the back for both methods, even though these ones have interchanged positions.

**Comparisons of the Methods**

The summarised orders appear below:

SAW $A_1, A_4, A_3, A_2, A_6, A_5$

WP $A_1, A_4, A_3, A_2, A_6, A_5$

TOPSIS $A_4, A_1, A_3, A_2, A_6, A_5$

NIS $A_1, A_3, A_2, A_4, A_6, A_5$

PIS $A_1, A_3, A_2, A_4, A_5, A_6$

The results of SAW and WP are completely identical. They reveal $A_1, A_4$ and $A_3$ (in this order) as the first three leading bidders. $A_5$ was at merit position four.

The results of TOPSIS were close to those of SAW and WP. The only small difference was for TOPSIS to interchange $A_1$ and $A_4$ by giving $A_4$ ahead of $A_1$. SAW, WP and TOPSIS gave the last three positions to $A_2, A_6$ and $A_3$ (in this order).

PIS and NIS gave the consistent results in the exercise used. They were consistent in that they gave as the first three ($A_2, A_3$ and $A_4$) as leading bidders. Their last bidders were interchanged in positions, but these are of no importance in this exercise. They also gave different results from the TOPSIS approach.

In PIS and NIS, $A_4$ went to merit position four after being at positions two (twice) and one and $A_5$ was advance slightly to position three after being at four.

TOPSIS seems to be one method which is closest to all of them by realising that it shares some similarities with SAW and WP where these differ with PIS and NIS. It may also seem to differ with the rest of them in that it is the method that shows to have most differences when bidders are placed from position one to six on an individual basis.

Being an aggregate of NIS and PIS, it should thus be considered ahead of them individually. In this case PIS and NIS were too close when used separately and the exercise does not provide adequate evidence to make a judgement.

All these methods agree that $A_1$ is a leading bidder, except that TOPSIS places it in the second position. TOPSIS has $A_4$ as the leading bidder. The problem is that PIS and NIS places $A_4$ in the fourth position.

The reason for not preferring PIS and NIS is that TOPSIS is a combination developed from them in order to adopt the positive aspects of both methods. Hence, even though they each show positive results when used separately, TOPSIS is the methods recommended together with SAW and WP [26].

**Concluding Discussion**
The descriptive distance measures/statistics were presented to demonstrate how they can be used together in selecting bidders or competitors when attributes and criteria are well-defined. Three methods (SAW, WP, TOPSIS) were used in placing preferred bidders. These methods showed consistency. When used together they stand to offset doubts that the chosen bidder was the deserving bidder. NIS and PIS may be used to support the results obtained when SAW, WP and TOPSIS were used bidding judgement. They were used in the exercise of this paper, and they showed some consistency with the three methods of this paper, but they still reveal doubts when use separately. They method they combined to form (TOPSIS), shows to be better than NIS and PIS separated.

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References

Appendix

Lemma

Recall the probability function \( p_i \) (and not that \( 0 \leq p_i \leq 1 \) and \( \sum_{i=1}^{n} p_i = 1 \)). Define

\[
E = -\sum_{i=1}^{n} p_i \ln p_i \quad (A1)
\]

Then
1. The minimum of \( E \) is \( E = 0 \).
2. The maximum of \( E \) is \( E = \ln n \).

Proof

Write \( E \) as \( E = \sum_{i=1}^{n} p_i \ln \frac{1}{p_i} \).

If \( p_i = 0 \), then \( \lim_{p_i \to 0} E = 0 \). That is, \( E = 0 \) when \( p_i = 0 \).

If \( p_i = 1 \), then \( \ln p_i = 0 \) so that \( E = 0 \).

Therefore, \( E = 0 \) at \( p_i = 0 \) and at \( p_i = 1 \).

Now, let \( 0 < p_i < 1 \). Then \( p_i > 0 \) and \( \ln \frac{1}{p_i} > 0 \).

Assume initially that the probabilities are all equal. That is \( p_i = \frac{1}{n} \). Then

\[
E = \sum_{i=1}^{n} \frac{1}{n} \ln n = \ln n.
\]

Now let \( p_i \) be the \( i \)th probability point. Granville et al. (1957) prove that for all real values of \( x > 0 \), the inequality

\[
\ln x \leq x - 1 \quad (A2)
\]

is valid.

Then,

\[
E - \ln n = \sum_{i=1}^{n} p_i \ln \frac{1}{p_i} - \sum_{i=1}^{n} p_i \ln n
\]

\[
= \sum_{i=1}^{n} p_i \left( \ln \frac{1}{p_i} - \ln n \right)
\]

\[
= \sum_{i=1}^{n} p_i \frac{1}{np_i}
\]

\[
\leq \sum_{i=1}^{n} p_i \left( \frac{1}{np_i} - 1 \right) \quad \text{(by inequality (A2))}
\]

\[
= \frac{1}{n} - \sum_{i=1}^{n} p_i
\]

\[
= 1 - 1 = 0.
\]

This gives \( E \leq \ln n \) and therefore

\( 0 \leq E \leq \ln n \).